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13. ABSTRACT (Maximum 200 words)

The project was devoted to analytic and numerical study of photonic band-gap materials. Among the results of the study are: development and implementation of an efficient finite element method for computing spectra of 2D acoustic and photonic PBG materials; development and numerical and analytic study of new asymptotic models for mesoscopic photonic, acoustic, semi-conducting, and super-conducting structures; study of novel possibilities of "localization" of light in purely periodic structures; study of novel geometries that allow opening of large spectral gaps; obtaining new analytic results about effects caused by localized impurities; study of properties of dispersion curves with respect to the properties of solutions.

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Final progress report for the DEPSCoR project "Mathematical analysis of photonic band-gap materials," 1997–2000

According to the proposal, the efforts of the PI P. Kuchment and of the Graduate Research Assistants W. Axmann, N. Benchama, L. Kunyansky, and H. Zeng were concentrated on the following problems:

- developing new asymptotic models of the problem in 2D and 3D;
- studying numerically and analytically the asymptotic models for different geometries of PBG structures in 2D and 3D;
- comparing asymptotic results with results for finite values of parameters;
- studying relations between spectral properties of photonic crystals and of other mesoscopic physical systems;
- using the asymptotic results together with Rayleigh-Ritz type methods for numerical estimating spectra of PBG structures;
- studying novel possibilities of "localization" of light in purely periodic structures;
- studying novel geometries that promise to open large spectral gaps;
- obtaining new analytic results about effects caused by localized impurities;
- implementing developed numerical methods into computer codes.

Significant progress has been achieved in all of these, as well as in some related directions.

Results achieved

An efficient finite element method for computing spectra in acoustic and 2D photonic cases was developed [1] and implemented. This method is applicable to arbitrary geometries of the media. It overcomes many shortcomings of the plane wave method. For instance, it easily takes care of discontinuities of the dielectric function of the medium and of the singular solutions that arise in high contrast media with thin dielectric walls (configuration which favors band gaps). This method has big advantage in comparison with the plane wave methods in terms of computer memory and speed required.

Graph models for mesoscopic super-conducting and photonic systems were developed ([15], [3], [8], [7], [2], [4]). These models lead to second and higher

order differential and also to first order pseudo-differential operators and spectral problems on graphs.

Numerical and mathematical analysis of these graph models was conducted ([8], [7], [2]). These models (first developed by the PI with coauthors) arise in the case when one considers high contrast photonic medium with thin dielectric walls. Similar models also arise in study of mesoscopic semi-conductor structures. The numerics discovered some unusual spectral behavior, which was explained then analytically. In particular, this analysis led to very much simplified approximate differential models on graphs, which produce in many cases amazingly good and easily computable approximations. An unusual effect (overlooked in previous physics studies of mesoscopic systems) of existence in some cases of bound states in purely periodic media was discovered.

The problem of absence of bounded states in a purely periodic acoustic or photonic medium was attacked ([10], [9]). In joint research with Prof. S. Levendorskii we have managed to develop a unifying approach to all such problems, which has already led to unification, advancement, and significant simplification of known results. It has also produced a simple model problem from complex analysis, which if resolved would resolve all remaining cases (including photonic and acoustic media) at once.

In spite of absence of bound states, it was shown [7] that special geometries can lead to extremely narrow spectral bands, i.e. to strong resonances. These could be used instead of bound states in certain applications like lasing or spontaneous emission enhancement.

The first available analytic result in several dimensions on absence of bounded states when on continuous spectrum for perturbed periodic media was obtained ([13], [14]).

A model was developed that allows opening band gaps using instead of periodicity presence of strong geometric scatterers in the medium [6]. This might hopefully lead to discovery of mechanisms of a much easier than usual opening of spectral gaps in 3D media.

New relations between near band edge analytic behavior of dispersion curves and Liouville type theorems for solutions was established [11].

Presentations and dissemination

During the project period, nineteen invited and four contributed talks at national and international meetings were presented. Among them were presentations at the Amer. Math. Soc. Meetings in Louisville, KY, Albuquerque, NM, Bolder, CO, Charlotte, NC, Birmingham, AL, and New Orleans, LA, at two international workshops in Austria and one in Israel, at the international conference on mathematical results in quantum mechanics in Prague, Czech Republic, a series of lectures at Ecole Polytechnique, France, and others.

Joint Summer Research Conference on mathematics of photonic crystals and related issues is planned for 2002.

The first survey of mathematical problems of photonic crystal theory was written [5].

Writing of a very much needed book on techniques for periodic PDEs with applications has been started [12].

Cooperation

We have cooperated with researchers from several academic institutions and industry in the USA and abroad, including researchers at UC Irvine, Texas A&M University, Duke University, University of North Carolina, WadeBand corp., Universities of Lyon and Toulon and Ecole Polytechnique(France), Technion (Israel), and University of Sussex (UK).

Educational impact

Four graduate students have been involved into the project:

Waldemar Axmann started work being a Masters student. Defended his Masters Thesis in July 1998. Currently in the physics PhD program at Kansas State University. Two papers published and one talk delivered at a national meeting.

Leonid Kunyansky defended his PhD Thesis in June 1998. The 1998 recipient of the Dora Wallace Hodgson Outstanding Doctoral Student Award. He is currently at Caltech at a postdoc position. Delivered several invited and contributed talks at national and regional meetings. Two papers published and one to be submitted soon.

Hongbiao Zeng is finishing his PhD dissertation in 2001. One presentation at a regional meeting, one paper accepted, and one more in preparation.

Noureddine Benchama is a beginning PhD student. One paper in progress.

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